4-6 Cramer's Rule

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- used determinants to solve systems of equations

Use elimination to solve for x:

$$d(ax + by = e)$$
$$-b(cx + dy = f)$$

$$adx + bdy = ed$$
$$-bcx - bdy = -bf$$
$$(ad-bc)x = de-bf$$
$$x = \frac{de-bf}{ad-bc}$$

$$x = \frac{de - bf}{ad - bc}$$

ax + by = e
cx + dy = f

Denominator is:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Write a determinant matrix for the numerator:

$$\begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

Cramer's Rule:

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$ax+by=e$$
$$cx+dy=f$$

Where D_x replaces the coefficients of x with the constants and D_y replaces the coefficients of y with the constants.

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

$$D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

ex:
x - 3y = 5
2x + 9y = -10

Replace x coeff. w/ 5 an -10 (constant)

$D = \begin{vmatrix} 1 & -3 \\ 2 & 9 \end{vmatrix} = 15$

$D_x = \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = 15$

$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -10 \end{vmatrix} = -20$

$x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$

$x = \frac{15}{15}$ $y = \frac{-20}{15}$ $(1, -\frac{4}{3})$

$x = 1$ $y = -\frac{4}{3}$

If D = 0

then, either

∞ # of solutions

OR

∅

Solve another way

ex:
2x - y = 3
6x - 3y = -9

$-6x + 3y = -9$
$6x - 3y = -9$
$0 = -18$
False

$D = \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = 0$

∅

3 Variables

$x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$ $z = \frac{D_z}{D}$

Cramer's Rule:

x = y = z =

ex:
4x + y - z = -2
x + 3y - 4z = 1
2x - y + 3z = 4

HW
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$$D_y = \begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -4 \\ 2 & 4 & 3 \end{vmatrix} = 96$$

$$D = \begin{vmatrix} 4 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 3 \end{vmatrix} = 16$$

$$D_z = \begin{vmatrix} 4 & 1 & -2 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{vmatrix} = 64$$

$$D_x = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & -1 & 3 \end{vmatrix} = -16$$

$$x = \frac{-16}{16} = -1 \qquad y = \frac{96}{16} = 6 \qquad z = \frac{64}{16} = 4$$